Cosmic Singularities
and
String Theory

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Plan

• What is a spacetime singularity?

• What is string theory?

• Two examples of stringy resolution of static singularities

• Cosmic singularities and string theory

• Lessons and implications
What is a spacetime singularity

- A spacetime is **singular** if there exists **at least** one freely falling particle or photon which ends (or has begun) its existence within a **finite** “time”.

- Intuitively, a **spacetime singularity** is a “place” where some “pathological behavior” of the metric takes place, e.g. the curvature “blows up” or …
Some Examples

- **Big Bang/Big Crunch**
  beginning or end of time?

- **Collapse of a star**
  black holes

- **Infinitely thin cosmic strings**
  Conical singularities
Singularities in general relativity

• In the late sixties, Hawking and Penrose showed that “generic” classes of spacetimes in general relativity are singular.

• Standard notion of spacetime breaks down at the singularities.

• General relativity and other known physics laws such as quantum field theory also break down.
Greatest crisis in physics of all time

-- “Gravitation” by Misner, Thorne and Wheeler

To understand singularities, one must go beyond the general relativity, e.g.

• Modify it at the classical level

• Quantum gravity
String Theory I

- **Quantum field theory**: particles are point-like objects propagating in space time.
- **String theory**: gravitons, photons and all other elementary particles are one-dimensional objects, strings.
- Interactions are described by splitting or joining of strings in spacetime.
- Point-particle description arises only in a low energy limit.

\[ \text{Tension} = \frac{1}{2 \pi \alpha'} \]
String theory II

• Presently, string theory is mostly formulated in terms of perturbation theory.
  – Perturbative degrees of freedom: graviton, Yang-Mills fields, …
  – Their masses have a well-defined limit as $g_s \to 0$.
• Much progress has been made toward a non-perturbative formulation.
  – Non-perturbative degrees of freedom: solitons, D-branes, black holes …
  – Their masses depend on inverse powers of $g_s$, and thus are not visible in perturbation theory.
  – M(atrix) theory, AdS/CFT ….
Example:

Graviton + Graviton \rightarrow Graviton + Graviton

\[ A = \lambda_2 \ g_s^2 + O(g_s^4) + \cdots \]
A consistent theory of gravity

In string theory, interactions no longer occur at points. It eliminates the UV problem in general relativity. Thus it is a promising candidate for a consistent theory of gravity.
Low energy expansions

\[ L_{\text{eff}} = \frac{1}{g_s^2 \alpha'} \left( R + \alpha' R^2 + \cdots \right) + \cdots \]

A double expansion in:

Energy \(2\alpha'\) and \(g_s^2\)

\[ M^2_{pl} = \frac{1}{g_s^2 \alpha'} \]

At low energy it reduces to Einstein relativity.
String theory and singularities

Can singularities in general relativity be resolved in string theory?

• in perturbative string theory (due to extended nature of strings)?

• Or does one need a non-perturbative formulation?
Conical Singularities

- Einstein gravity breaks down at a generic conical singularity. Those time- or null-like geodesics which hit the singularity cannot be continued beyond.
Orbifolds

• For those obtainable from 2-dimensional flat space by *discrete identifications*, one can still make sense the classical theory, e.g.

\[(x_1, x_2) \sim (-x_1, -x_2)\]

\[\mathbb{R}^2/\mathbb{Z}_2\]
Orbifolds II

• Similarly, one can try to define a quantum field theory in the cone by projecting onto the subspace of the Hilbert space in $\mathbb{R}^2$ which are invariant under the identifications.

• However, a field theory defined this way is not unitary at the quantum level.
String theory on orbifolds

- The extended nature of string theory introduces additional degrees of freedom localized at the tip of the cone: twisted sectors.

- Including the twisted sectors, string amplitudes are unitary and physics is completely smooth.

- This is an example where string theory resolves the singularity at the perturbative level.
Conifold

- General relativity is *singular*.
- Classical string theory is *singular*.
- By including the non-perturbative degrees of freedom at the tip of the cone, the physics is again *smooth*.

![Diagram of Conifold]
Lessons

• String theory introduces new degrees of freedom. By including them the physics at those singularities becomes completely smooth.

• Those singularities arise in general relativity simply because the relevant degrees of freedom are not visible.
Cosmological singularities

• Beginning of time
  – Need initial conditions, wave functions of the Universe etc.

• Time has no beginning or end
  – Need to understand how to pass through the singularity. ........
Challenges for string theory

• **Hope**
  – String theory will lead to a detailed theory of the Big Bang.
  – Experimental tests of string theory.

• **Question**
  Is the cosmological singularity smoothed by *classical string theory* or *quantum string theory*?
From Big Crunch to Big Bang: is it possible?
(A Toy Model)

Time-dependent orbifolds: obtainable from discrete identifications of a flat Lorentzian spacetime. (Horowitz, Steif)
Motivations

• Basis for some recently proposed cosmological scenarios: Ekpyrotic/Cyclic Model. Khoury, Ovrut, Seiberg, Steinhardt, Turok

• Simplicity: it can be subjected to an exact perturbative string analysis.

• Universality: the structure of singularity is the same as in certain black holes and certain more complicated cosmological backgrounds.
Results from string perturbation theory

Liu, Moore, Seiberg

- One can compute the **S-matrix** from one cone to the other.

- For *generic* kinematics the amplitudes in classical string theory are **finite** (while they diverge in GR). This may be attributed to the **softness** of strings at high energies.

- For special kinematics (**near forward scattering**) the string amplitudes **diverge**.
Origin of the divergence

- Since the background depends on time, energy is not conserved.
- The energy of an incoming particle is blue shifted to infinity by the contraction at the singularity.
- The infinite energy generates infinitely large gravitational field and distorts the geometry.
- String perturbative expansion breaks down as a result of large backreaction.
Lessons

• Classical string theory is singular in time dependent singular orbifolds.

• The extended nature of strings is not sufficient to resolve the singularity.

• Need to understand the full (non-perturbative) quantum theory to explain the physics at the singularity.
Imagination for a non-singular bounce?

• The idea of going from a big crunch to a big bang through a non-singular bounce has a long history:
  
  30’s Einstein, Tolman ……

• The singularity theorems of Hawking and Penrose ruled out this possibility in general relativity.

• The recent suggestions that the universe passes through the singularity is motivated by the orbifold construction of string theory.

• We now see that classical string theory is also singular and cannot be trusted.
Summary

• String theory is a promising candidate for a consistent theory of quantum gravity.

• Certain singularities in GR are resolved in perturbative string theory, while others are resolved by invoking non-perturbative degrees of freedom.

• Understanding the cosmic singularities is a big challenge for string theory. String theory has the potential to make important progress in cosmology by addressing this question.

• Our investigation indicates one needs to develop new non-perturbative tools to solve this problem.