Dephasing, decoherence, 

What is it all about?

Consider a spinor (total phase is irrelevant)

Average components of the spin can be expressed through the absolute values of the spinor's components and the phase

Let us start with $S_z = 0$, $\theta = 0$, i.e. $\vec{S} \parallel \vec{x}$ and apply magnetic field $\vec{B} \parallel \vec{z}$ for some time interval $t$. Spin rotation and apply magnetic field $\vec{B} \parallel \vec{z}$ for some time interval $t$. Spin rotation

Coherent Oscillations of, e.g., $x$-projection of the spin (!?)
Let us start with $S_z = 0$, $\square = 0$, i.e. $\vec{S} \parallel \vec{x}$ and apply magnetic field $\vec{B} \parallel \vec{z}$ for some time interval $\square t$. Spin rotation $\square \square \vec{B} \square \square t$.

Coherent Oscillations of, e.g., $x$-projection of the spin (?!)

One can also apply time-independent field $\vec{B} \parallel \vec{x}$, that will cause the Zeeman splitting, and also a small oscillating in time field in $\vec{z}$ direction, with the frequency close to the Zeeman splitting. There will be again rotation in $xy$ -plane $\square$ Rabi oscillations.

Decoherence of a single spin

Consider a spinor (total phase is irrelevant)

Average components of the spin can be expressed through the absolute values of the spinor's components and the phase

A random time-dependent magnetic field $\vec{B}(t)$ changes the phase in an uncontrollable way $\leftrightarrow$ it rotates the spin.

This can be called **decoherence**

**Relaxation times** $T_1, T_2$
Dephasing and Decoherence: What's It All About?

Coherent control of macroscopic states in a single-Cooper-pair box

Y. Nakamura, Yu. A. Pashkin, & J. S. Tsai

A nanometre-scale superconducting electrode connected to a reservoir via a Josephson junction constitutes an artificial two-level electronic system: a single-Cooper-pair box. The two levels consist of charge states (differing by $2e$, where $e$ is the electronic charge) that are coupled by tunnelling of Cooper pairs through the junction. Although the two-level system is macroscopic, containing a large number of electrons, the two charge states can be coherently superposed\(^1\)–\(^4\), the basic component of a quantum computer. Here we report the observation of quantum oscillations in a single-Cooper-pair box. By applying a short voltage pulse via a gate electrode, we can control the coherent quantum state evolution; the pulse modifies the energies of the two charge states nonadiabatically, bringing them into resonance. The resulting state—a superposition of the two charge states—is detected by a tunnelling current through a probe junction. Our results demonstrate electrical coherent control of a qubit in a solid-state electronic device.

Figure 1: Single-Cooper-pair box with a probe junction. a, Micrograph of the sample. The electrodes were fabricated by electron-beam lithography and shadow evaporation of Al on a Si$_3$N$_4$ insulating layer (400 nm thick) above a gold ground plane (100 nm thick) on the oxidized Si substrate. The 'box' electrode is a $73 \times 130 \times 15$ nm Al layer containing 19 conduction electrons. The reservoir electrode was evaporated after a slight oxidation of the surface of the box so that the overlapping area becomes two parallel low-resistance tunnel junctions (total) with Josephson energy $E_J$, which can be tuned through magnetic flux $\Phi$ penetrating through the loop. Before the evaporation of the probe electrode we further oxidized the box to create a highly resistive probe junction ($|r| > 3 \times 10^3$). Two gate electrodes (dc and pulse) are capacitively coupled to the box electrode. The sample was placed in a shielded cryostat at the base temperature of 77 K (liquid nitrogen) of a dilution refrigerator. The single-electron charging energy of the box electrode $E_C = \frac{e^2}{2C}$, was $17 \pm 3 \mu$V, where $C$ is the total capacitance of the box electrode. The superconducting gap energy $E_g$ was $200 \pm 10 \mu$V. The circuit diagram of the device. The $C_b$ represents the capacitance of each element and the $V_b$ is the voltage applied to each electrode.
Dephasing and Decoherence: What's It All About?

Figure 2: Pulse modulation of quantum states. a. Energy diagram illustrating electronic energies (solid lines) of two charge states (1 and 2) as a function of two gate-induced charge $Q_1 = Q_{	ext{g}} + Q_{	ext{gate}}$, where $Q_{	ext{gate}}$ is the gate-induced charge. The dashed curves show energies (in the absence of the quadrupole-particle tunneling at the probe junction) as a function of $Q_{	ext{gate}}$. Suppose that below a pulse occurs, $Q_{	ext{g}} = Q_{	ext{g}}^*$, which is far from the resonance point, and the system is approximately in the pure charge state $|0\rangle$ (filled circle at lower left). Then, a voltage pulse of an appropriate height abruptly brings the system into resonance $Q_{	ext{g}} = 1$ (solid arrow), and the state starts to oscillate between the two charge states. At the end of the pulse, the system returns to $Q_{	ext{g}} = Q_{	ext{g}}^*$ (dashed arrow) with a final state corresponding to the result of the time evolution. Finally, the 2 state decays to $|0\rangle$ with two quadrupole tunneling events through the probe junction with rates of $I_{1/2}$ and $I_{1/2}$ (dotted arrows). b. Schematic pulse shape with a nominal pulse length at solid line. The effective turn-on of the actual voltage pulse was about 30-40 ps at the top of the data. The voltage pulse was transmitted through a silver-plated Be-O Cu coaxial cable (above 2 K), a Nb coaxial cable (below 2 K) and an on-chip planar line to the open-ended pulse gate shown in Fig. 1a. The inset illustrates situations of the energy levels before and during a square pulse.

Dr. Boris Altshuler, KITP, Princeton, & NEC Research Inst (KITP Glassy States 4/08/03)
Quantum oscillations in two coupled charge qubits


Figure 1. Two capacitively coupled charge qubits. a. Scanning electron micrograph of the sample. The qubits were fabricated by electron-beam lithography and three-angle evaporation of Al (light areas) on a SiN$_x$ insulating layer (dark) (see ref.5 for fabrication details). Two qubits are coupled by an additional coupling island overlapping both Cooper-pair boxes. Although the coupling island has a finite tunneling resistance $\sim 10 \, \text{M} \Omega$ to the boxes, we consider the coupling as purely capacitive represented by a single capacitor in the equivalent circuit because all the tunneling processes are completely blocked. The estimated capacitance of the island to the ground is $\sim 1 \, \text{aF}$. b. Equivalent circuit of the device. The parameters obtained from the dc measurements are: $C_{11} = 620 \, \text{aF}, C_{12} = 469 \, \text{aF}, C_{13} = 11 \, \text{aF}, C_{14} = 50 \, \text{aF}, C_{21} = 0.60 \, \text{aF}, C_{22} = 0.61 \, \text{aF}, C_{31} = 1 \, \text{aF}, C_{32} = 34 \, \text{aF}$, and the corresponding energies are $E_{C1} = 484 \, \mu \text{eV}$ (117 GHz in frequency units), $E_{C2} = 628 \, \mu \text{eV}$ (152 GHz) and $E_{C3} = 65 \, \mu \text{eV}$ (15.7 GHz). Josephson coupling energies, $E_{J1} = 55 \, \mu \text{eV}$ (13.4 GHz) and $E_{J2} = 38 \, \mu \text{eV}$ (9.1 GHz), were determined from the single qubit measurements described later in the text. Probe junction tunnel resistance is equal to 31.6 $\, \text{M} \Omega$ (left) and 34.5 $\, \text{M} \Omega$ (right). Superconducting energy gap is 210 $\mu \text{eV}$. Black bars denote Cooper-pair boxes. Symbol $\bigoplus$ represents a tunnel junction without Josephson coupling, while $\bigotimes$ is a Josephson tunnel junction.
Dephasing and Decoherence: What's It All About?

Quantum oscillations in two coupled charge qubits


Figure 2. Pulse operation of the device. a. Schematics of the ground-state changing diagram of the coupled qubits as a function of the normalised gate charges $n_1$ and $n_2$. The number of Cooper pairs $n_1$ and $n_2$ in the neighboring cells differs by one. The electrostatic energies $E_{\text{elec}}$ are degenerate at the boundaries. Points R and L correspond to energy degeneracy in the first and the second qubit, respectively. Point X is doubly degenerate: $E_{\text{X}} = E_{\text{R}}$ and $E_{\text{X}} = E_{\text{L}}$. Arrows show how pulses shift the system in the experiment. b. Energy diagram of the system along the line $n_1 = n_2$ through the point X. Solid lines are the electrostatic energies of charge states $|00\rangle$, $|10\rangle$, $|01\rangle$ and $|11\rangle$. Dashed lines are eigenenergies of the Hamiltonian (1). Far from co-resonance (point X in a), the system stays in $|00\rangle$. After the pulse brings the system to the co-resonance (solid arrow), the system starts to evolve producing a superposed state $|\psi(0)\rangle = c_1|00\rangle + c_2|10\rangle + c_3|01\rangle + c_4|11\rangle$. The amplitudes $|c_i|^2$ ($i = 1, 2, 3, 4$) remain “frozen” after the pulse termination (dashed arrows) until the resulting state decays into the ground state. The decay process indicated by grey arrows contributes to the probe currents proportional to the probabilities (3).

Quantum oscillations in two coupled charge qubits


Figure 3. Quantum oscillations in qubits. a. Probe current oscillations in the first (top) and the second (bottom) qubit, when the system is driven to the points R and L, respectively. Right panel shows corresponding spectra obtained by the Fourier transform. In both cases, the experimental data (open triangles and open dots) can be fitted to a cosine dependence (solid lines) with an exponential decay with $2\tau_{\text{ec}}$ as time constant. b. Probe current oscillations in the qubits at the co-resonance point X. Their spectra (right panel) contain two components. Arrows and dotted lines indicate the position of $\Omega + \epsilon$ and $\Omega - \epsilon$ obtained from $\Omega$ using $E_{\text{F1}} = 13.4$ GHz, $E_{\text{F2}} = 9.1$ GHz measured in the single qubit experiments (Fig. 5a) and $E_{\text{F1}} = 15.7$ GHz estimated independently from the measurements. Solid lines are fits obtained from numerical simulation with the parameters $E_{\text{F1}} = 13.4$ GHz, $E_{\text{F2}} = 9.1$ GHz, and $E_{\text{F1}} = 14.5$ GHz. Finite pulse rise/fall time and not pure $|00\rangle$ initial condition were taken into account. The introduced exponential decay time is $0.6 \text{ ns}$.
1. Suppose that originally a system (an electron) was in a **pure** quantum state. It means that it could be described by a **wave function** with a given **phase**.

2. External perturbations can transfer the system to a different quantum state. Such a transition is characterized by its **amplitude**, which has a **modulus** and a **phase**.

3. The **phase** of the **amplitude** can be measured by comparing it with the **phase** of another amplitude of the same transition. **Example**: Fabri-Perrot interferometer

4. Usually we **can not** control all of the perturbations. As a result, even for fixed initial and final states, the **phase** of the transition amplitude has a **random** component.

5. We call this contribution to the **phase**, \( \Phi \), **random** if it changes from measurement to measurement in an uncontrollable way.

6. It usually also depends on the duration of the experiment, \( t \):

\[
\Phi = \Phi(t)
\]

7. When the time \( t \) is large enough, \( \Phi \) exceeds \( 2\pi \), and interference gets averaged out.

8. **Definitions**:

\[
\frac{\Phi(t)}{\Phi}  \leq 2\pi
\]

\( \tau \) **phase coherence time**; \( \frac{1}{\Gamma} \) **dephasing rate**
Why is Dephasing rate important?

Imagine that we need to measure the energy of a quantum system, which interacts with an environment and can exchange energy with it.

Let the typical energy transferred between our system and the environment in time $t$ be $\Delta(t)$. The total uncertainty of an ideal measurement is

$$\Delta(t) \Delta(t) + \frac{\hbar}{t}$$

There should be an optimal measurement time $t = t^*$, which minimizes $\Delta(t)$:

$$\Delta(t^*) = \Delta_{\text{min}} \quad t^* = \frac{\hbar}{\Delta_{\text{min}}}$$

It is dephasing rate that determines the accuracy at which the energy of the quantum state can be measured in principle.
Dephasing and Decoherence: What's It All About?

**How to detect phase coherence measure the dephasing/decoherence rate?**

Quantum phenomena in electronic systems:
- Weak localization
- Mesoscopic fluctuations
- Persistent current
- Orthogonality catastrophe

**Magnetoresistance**

No magnetic field

\[ \text{No magnetic field} \]

\[ j_1 = j_2 \]

With magnetic field \( H \)

\[ j_1 - j_2 = \frac{2\pi}{F} \]

\[ F = HS - \text{magnetic flux through the loop} \]

\[ F_0 = \frac{hc}{e} - \text{flux quantum} \]
Dephasing and Decoherence: What's It All About?

**Weak Localization, Magnetoresistance in Metallic Wires**

**Mesoscopic Fluctuations; Aharonov-Bohm effect**

Can we reliably extract the dephasing rate from the experiment?

Is energy transfer necessary for the “dephasing”?

**Weak localization:**

NO - everything that violates $T$-invariance will destroy the constructive interference

EXAMPLE: random quenched magnetic field

**Mesoscopic fluctuations:**

YES - Even strong magnetic field will not eliminate these fluctuations. It will only reduce their amplitude by factor 2.

Therefore

Statistical analysis is unavoidable if we want to experimentally determine the dephasing rate.
Dephasing and Decoherence: What's It All About?

**On the other hand**

Let the random potential change in time very slowly, but still quick on the scale of measurement time.

**Weak localization:**

- Averaging over different realizations of the disorder.

- At any moment $j_1 = j_2$.

**Mesoscopic fluctuations:**

- Averaging over time.

- Is time dependent.

**Therefore**

Weak localization effects do not feel the motion of impurities.

Mesoscopic fluctuations get suppressed.

---

**Magnetic Impurities**

- Before and after.

- T-invariance is clearly violated, therefore we have dephasing.

**Mesoscopic fluctuations**

Magnetic impurities cause dephasing only through effective interaction between the electrons.

$0 \rightarrow T$

- Either Kondo scattering or quenching due to the RKKY exchange.

- In both cases no "elastic dephasing".

Long time (spin) dynamics in metallic (spin) glasses?
A spin-echo-type technique is applied to an artificial two-level system that utilizes a charge degree of freedom in a small superconducting electrode. Gate-voltage pulses are used to produce the necessary pulse sequence in order to eliminate the inhomogeneity effect in the time-ensemble measurement and to obtain refocused echo signals. Comparison of the decay time of the observed echo signal with an estimated decoherence time suggests that low-frequency energy-level fluctuations due to the $1/f$ charge noise dominate the dephasing in the system.

**FIG. 1.** (a) Schematic of a Cooper-pair box with an additional probe electrode. (b) Bloch sphere representations of schematic quantum-state evolutions at $Q_0 = 0.45e$ corresponding to the two pulse heights $\Delta Q_p = 0.55e$ (top) and $\Delta Q_p = 0.53e$ (bottom). The thin arrow in the $xz$-plane indicates the direction of the effective magnetic field.
Dephasing and Decoherence: What's It All About?

FIG. 2. Charge-echo experiment: (a) Schematic quantum-state evolutions. (b) Pulse sequence. (c) Normalized free-induction decay (FID) signal vs. $\delta t_3$ taken without the second pulse and with $t_2 = 0$. The oscillating signal is highpass-filtered and normalized to the gaussian envelope $\exp(-\left(\delta t_3/150\text{ps}\right)^2)$. (d) Normalized echo signal vs. $\delta t_3$. The envelope is $\exp(-\left(\delta t_3/100\text{ps}\right)^2)$. The signal-to-noise ratio is poor, because the data was taken with three $\pi/2$-pulses instead of the ideal pulse sequence. $T_1$ is 64 ns in (c) and (d). (e) Echo-signal current $I$ vs. $\delta t_2$. Solid curve is a sinusoidal fit. (f) Oscillation amplitude of the echo-signal current $\Delta I$ as a function of $2\delta t_2$ with a gaussian fit.

Decoherence factor

Let the spacing between the two levels, $E$, fluctuate in time:

random phase

$D(\theta) = \frac{1}{\hbar} \int_0^\Delta D(\theta) d\theta \quad \Rightarrow \quad e^{iD(\theta)} = \langle e^{i\theta} \rangle$

Assume that $\langle \theta \rangle$ is gaussian, and

$\langle \theta \theta \rangle_{\Delta} = s_{\Delta E} \langle \theta \rangle$

$D(\theta) = \frac{1}{2\hbar^2} \int_{-\infty}^{\infty} \langle \theta \theta \rangle \sin(\theta/2) \sin(\theta/2) d\theta$

$m$ is inverse time of the measurement
Dephasing and Decoherence: What's It All About?

**Decoherence factor**

\[
\frac{I_{\text{max}}(\theta)}{I_{\text{max}}(0)} = e^{D(\theta)}
\]

For the free induction decay

\[
D_{\text{FID}}(\theta) = \frac{1}{2\hbar^2} \int_{\Delta m} E^2(\theta) \left[ \frac{\sin(\theta/2)}{\theta/2} \right]^2 d\theta
\]

For the echo signal

\[
D_{\text{echo}}(\theta) = \frac{1}{2\hbar^2} \int_{\Delta m} E^2(\theta) \left[ \frac{\sin^2(\theta/4)}{\theta/4} \right]^2 d\theta
\]

At \( \theta = 0 \)

\[
\frac{\sin(\theta/2)}{\theta/2} \quad \text{const} \quad \frac{\sin^2(\theta/4)}{\theta/4}
\]

Even \( \theta << \Delta m \) are important

Low frequency fluctuations are not important

**Decoherence factor**

\[
\frac{I_{\text{max}}(\theta)}{I_{\text{max}}(0)} = e^{D(\theta)}
\]

\[
D_{\text{FID}}(\theta) = \frac{1}{2\hbar^2} \int_{\Delta m} E^2(\theta) \left[ \frac{\sin(\theta/2)}{\theta/2} \right]^2 d\theta
\]

\[
D_{\text{echo}}(\theta) = \frac{1}{2\hbar^2} \int_{\Delta m} E^2(\theta) \left[ \frac{\sin^2(\theta/4)}{\theta/4} \right]^2 d\theta
\]

\[
\langle \theta(\theta) \theta(\theta) \rangle = S_{\Delta E}(\theta)
\]

\[
D_{\text{FID}}(\theta) \quad \theta^2 \quad \log \frac{1}{\Delta m}
\]

\[
D_{\text{echo}}(\theta) \quad \theta^2
\]

\[
\log(10^9) \gg 1
\]

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Dephasing and Decoherence: What's It All About?

It is not a true decoherence. It is rather a temporally inhomogeneous broadening:

Different shots cause oscillations with slightly different frequencies

The echo technique allows one to get rid of the temporally inhomogeneous broadening:

\[ D_{\text{echo}}(\tau) = \frac{\sin^2(\frac{\tau}{4})}{\frac{\tau}{4}} d\tau \]

This integral is determined by \( \tau \) of the order of \( \frac{\tau}{4} \)
Decoherence factor

Let the spacing between the two levels, $\Delta E$, fluctuate in time:

$$I_{\text{max}}(\Delta t) = e^{-\Delta D(\Delta t)}$$

$$\Delta D(\Delta t) = \Delta E \Delta \langle \Delta E \rangle$$

Gate error

What is the origin of the $1/f$ noise?

1. One fluctuator (e.g., 2 level system) - telegraph noise. Time-dependent dipole moment $d(t)$:

$$\langle d(t) \rangle \neq \exp(\Delta \Delta t)$$

2. Several fluctuators with different values of the relaxation rate $\Gamma$:

$$\langle \Delta \rho(t) \rangle_{\Delta \Gamma} \neq \sum_{i} C_i \Delta \Gamma_i^{\frac{1}{2}} + \Delta \Gamma$$

$C_i$ is the coupling constant of the $i$-th fluctuator, and $\langle C \rangle$ is its mean value. $P(\Gamma)$ is the probability density of the relaxation rates. We assumed that $C_i$ and $\Gamma$ are statistically independent.

3. Assuming now that $\log(\Gamma)$ is distributed uniformly (it is natural if $\exp(\Delta S)$, i.e., $P(\Gamma)$ $1/\Gamma$ we obtain

$$\langle \Delta \rho(t) \rangle_{\Delta \Gamma} \neq \frac{1}{\Delta \Gamma}$$
Dephasing and Decoherence: What's It All About?

“Fluctuations in the intensity of 1/f noise in disordered metals”

\[ \langle \delta \phi(t) \delta \phi(0) \rangle = \frac{C_i}{\Omega_i^2 + \Omega^2} + \frac{\Omega}{\Omega_i} \mu_i \]

For a small system (qubit) each contribution, \( \Omega_i \), is inverse proportional to the cube of the distance, \( r \), between the system and the fluctuating dipole moment:

\[ \Omega_i \propto r^{-3} \]

The decoherence is dominated by a single fluctuator.

Telegraph noise

\[ D(\Omega) \]

\( D(\Omega) \propto \Omega^2 \)

Different fluctuators dominate at different scales.

\[ D(\Omega) \propto \Omega^2 \]

1/f noise

\[ D(\Omega) \propto \Omega^2 \]

The decoherence is dominated by a single fluctuator.
Quantum oscillations in two coupled charge qubits


Figure 3. Quantum oscillations in qubits. a. Probe current oscillations in the first (top) and the second (bottom) qubit when the system is driven to the points R and L, respectively. Right panel shows corresponding spectra obtained by the Fourier transform. In both cases, the experimental data (open triangles and open dots) can be fitted to a cosine dependence (solid lines) with an exponential decay with 2.5 ns time constant. b. Probe current oscillations in the qubits at the co-resonance point X. The spectra (right panel) contain two components. Arrows and dotted lines indicate the position of $\Omega + \epsilon$, $\Omega - \epsilon$ obtained from $\Omega$ using $\Omega_0 = 13.4$ GHz, $\Omega_{01} = 9.1$ GHz measured in the single qubit experiments (Fig. 3a) and $\Omega_{02} = 15.7$ GHz estimated independently from dc measurements. Solid lines are fits obtained from numerical simulation with the parameters $\Omega_0 = 13.4$ GHz, $\Omega_{01} = 9.1$ GHz and $\Omega_{02} = 14.5$ GHz. Finite pulse rise/fall time and not pure 00- initial condition were taken into account. The introduced exponential decay time is 0.6 ns.
Dephasing and Decoherence: What's It All About?

Quantum oscillations in two coupled charge qubits

Yu. A. Pashkin, T. Yamamoto, O. Astafiev,
Y. Nakamura, D. V. Averin & J. S. Tsai

![Diagram](image)

Schematics of the ground-state changing diagram of the coupled qubits as a function of the normalised gate charges \( n_g \) and \( n_F \). The number of Cooper pairs \( n_1 \) and \( n_2 \) in the neighbouring cells differs by one. The electrostatic energies \( E_{\text{cou}} \) are degenerate at the boundaries. Points \( R \) and \( L \) correspond to energy degeneracy in the first and the second qubit, respectively. Point \( X \) is doubly degenerate: \( E_{\text{q1}} = E_{\text{q2}} \) and \( E_{\text{q1}} = E_{\text{q2}} \). Arrows show how pulses shift the system in the experiment.

Quantum oscillations in two coupled charge qubits

Yu. A. Pashkin, T. Yamamoto, O. Astafiev,
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![Graphs](image)

\[
(0,0) [(1,0)] \\
+ \\
(0,0) [(0,1)] \\
\text{is not distinguished from} \\
(0,0) [(1,1)]
\]
Dephasing and Decoherence: What's It All About?

Consider a spinor (total phase is irrelevant)

\[ Y = y \uparrow e^{ijy} \]

Average components of the spin can be expressed through the absolute values of the spinor’s components and the phase

\[ S_x = \sqrt{1 - S_y^2 \cos} \quad S_y = \sqrt{1 - S_z^2 \sin} \]

A random time-dependent magnetic field \( \vec{B}(t) \) changes the phase in an uncontrollable way — it rotates the spin.

\[ \vec{S} \]

This can be called decoherence

However

A classical magnet also gets rotated by external magnetic fields

\[ \vec{B} \]

For the quantum computation two-level systems \((S=1/2)\) are necessary. Involvement of highly excited states is not permissible.
Dephasing and Decoherence: What's It All About?

Decoherence

Inelastic Dephasing

Gate errors

Leakage

Interaction with thermal fluctuations of the (equilibrium) environment

Spontaneous relaxation of an excited quantum system with emission of excitations in the environment
Dephasing and Decoherence: What's It All About?

Inelastic dephasing

- other electrons
- phonons
- magnons
- two level systems

Can be modeled, e.g., by an interaction with an oscillator bath

Persistent current

\[ J \frac{\partial E}{\partial \phi} \]

\( E \) is the ground state energy

Completely quantum phenomenon !?
Persistent current at zero temperature is a property of the ground state!

\[ J = \frac{\partial E}{\partial \mu} \]

\( E \) is the ground state energy

Interaction between electrons can change both \( E \) and \( J \), but this does not mean that there is dephasing of the ground state wave function.

Measurements of the persistent current as well as of other thermodynamic properties do not allow to extract the dephasing rate.

We prove that the ground state of a system of \( N \) fermions is to the ground state in the presence of a finite range scattering potential, as \( N \uparrow \). This implies that the response to application of such a potential involves only emission of excitations into the continuum, and that certain processes in Fermi gases may be blocked by orthogonality in a low - \( T \), low - energy limit.
Dephasing and Decoherence: What's It All About?

Interaction with the environment suppresses the tunneling, i.e., the splitting.
Is it a dephasing?

Typical problem/mistake:

Quantum System interaction Environment

Nobody yet invented device that can universally serve as a “phasometer”.

\[ |I\rangle = |0\rangle e^{iQ} \]
Dephasing and Decoherence: What's It All About?

\[ |I_i \rangle = |I_i \rangle e^{i/2} \]
\[ |II_i \rangle = |II_i \rangle e^{i/2} \]
\[ |\text{tot} \rangle = |\text{tot} \rangle e^{i/2} \]
\[ (\hat{H}_I + \hat{H}_{II} + \hat{H}_{\text{int}}) |\text{tot} \rangle = E |\text{tot} \rangle \]

Ground state:
\[ |0_{\text{tot}} \rangle \]
\[ |0_{\text{tot}} \rangle \neq |0_I \rangle |0_{II} \rangle \]

is not even an eigenstate of the system. It is a complex superposition of excited states. Being prepared in this particular state the system starts to evolve in time, and this evolution can look like a phase relaxation of, say, the system I.
Dephasing and Decoherence: What's It All About?

**Quantum System I**  
**Quantum System II**

**Ground state:**  
\[ 0^\text{tot} \]

is not even an eigenstate of the system. It is a complex superposition of excited states. Being prepared in this particular state the system starts to evolve in time, and this evolution can look like a phase relaxation of, say, the system I.

**Q:** Does it mean that there is zero temperature dephasing?  
**A:** No

**Inelastic dephasing**

- other electrons
- phonons
- magnons
- two level systems

Can be modeled, e.g., by an interaction with an oscillator bath
Dephasing and Decoherence: What's It All About?

**e-e interaction – Electric noise**

**Fluctuation- dissipation theorem:**

Electric noise - randomly time and space - dependent electric field \( E^\pi(\vec{r},t) \) \( E^\pi(\vec{k},0) \). Correlation function of this field is completely determined by the conductivity \( \sigma(\vec{k},0) \):

\[
\langle E^\pi E^\pi \rangle_{\omega,k} = \frac{k^2}{2T} \coth \frac{k_r}{2T} \sigma(\vec{k},0) \frac{T}{\sigma(\vec{k},0)}
\]

*Noise intensity increases with the temperature, \( T \), and with resistance*

**Dephasing rate due to e-e interaction for 1d and 2d cases**

\[
\langle E^\pi E^\pi \rangle_{\omega,k} = \frac{k^2}{2T} \coth \frac{k_r}{2T} \sigma(\vec{k},0) \frac{T}{\sigma(\vec{k},0)}
\]

\( g(L) = \frac{\hbar}{e^2 R(L)} \) - Thouless conductance – def.

\( R(L) \) - resistance of the sample with \( \{ \text{length (1d)}, \text{area (2d)} \} \) \( L \)

\[
\frac{1}{g(L)} = \frac{T}{\sigma(\vec{k},0)}
\]

\( L_\square = \sqrt{D \square} \) - dephasing length \( D \) - diffusion constant of the electrons
Dephasing and Decoherence: What's It All About?

Fermi liquid is valid (one particle excitations are well defined), provided that
\[ T \frac{1}{\mathcal{L}_g(L)} > \hat{\hbar} \]

1. In a purely 1d chain, \( g \frac{1}{\mathcal{L}_g} \), and therefore, Fermi liquid theory is never valid.

2. In a multichannel wire \( g(L_{\mathcal{L}}) > 1 \), provided that \( L_{\mathcal{L}} \) is smaller than the localization length, and Fermi liquid approach is justified.

\[ T \frac{1}{\mathcal{L}_g(L)} = \frac{T}{\mathcal{L}_g} \]

\[ g(L) \rightleftharpoons L^{d_{\mathcal{L}}^2} \]

where \( d \) is the number of dimensions:
- \( d=1 \) for wires;
- \( d=2 \) for films, ...

\[ L_{\mathcal{L}} \rightleftharpoons \sqrt{\mathcal{L}_g} \]

\[ L_{\mathcal{L}} \rightleftharpoons T^{1/(4d_{\mathcal{L}})} \]

\[ \mathcal{L}_g \rightleftharpoons T^{1/d_{\mathcal{L}}} \]

\[ T^{1/3} \]

\[ T^{1/3} \]

\[ T^{2/3} \]