Event-by-event fluctuations – theoretical perspective

- Two ideas:
  - QCD critical point (MS, Rajagopal, Shuryak)
  - Frozen QGP charge fluctuations (Asakawa, Heinz, Muller, Jeon, Koch)

- Framework

1. Lattice calculations: no singularity at $\mu = 0$ (crossover).
3. $1 + 2 = 3$: The 1st order transition must end at critical end-point $E$. As in water at $p = 221$bar, $T = 373°C$ – critical opalescence.

Where is point $E$? Challenge for theory and experiment.

Heavy ion collision experiments can discover $E$ and leave a mark on the phase diagram of QCD.
Heavy ion collisions and the QCD phase diagram

"Little Bang"

Time history of a small macroscopic subvolume:

\[ \tau_\text{H} \sim 10 - 20 \text{ fm}; \]
\[ \tau_\text{free} \sim 1 \text{ fm}; \]
grows as \( n \to 0 \)
when \( \tau_\text{free} \sim \tau_\text{H} - \text{freezeout} \)

(CMB)

Observed hadron spectra reflect thermodynamics at the time of "last interaction" — freezeout time

(Braun-Munzinger, Stachel) \( \rightarrow T, \mu \) at freezeout

Strategy: scan the phase diagram changing \( \sqrt{s} \).
• Why event-by-event fluctuations?

Criticality is always due to a divergent correlation length (= vanishing mass).

In QCD it is \( m_\sigma \to 0 \) (\( \sigma \) - fluctuation of the magnitude of \( \langle \bar{\psi}\psi \rangle \))

\( \sigma \)'s we do not see after freezeout, because \( \sigma \to \pi\pi \) in vacuum

However, at freezeout, fluctuations of the \( \sigma \) field (\( \sim 1/m_\sigma^2 \)) create correlations in the pion momenta distributions (due to \( \sigma\pi\pi \) coupling)

![Diagram showing correlation between pions and \( \sigma \) field]

Such correlations can be measured using e-b-e fluctuations (of \( p_T \))
Misha Stephanov, Univ. Illinois (ITP QCD-RHIC Conference 4-12-02) Fluctuations: theory

Q: What distinguishes σ contribution to F from possible other contrasts?

A: NONMONOTONIC BEHAVIOR ON VS (σ parameter controlling "hypercenter")

\[ \langle \Delta n_p \Delta n_{\pi} \rangle \sim \frac{\Delta p_T \Delta k_T}{\sigma_{inc}^2} \]

Define a correlator:

\[ C_{p_T}(y_1, y_2) = \sum \delta(y_1 - y_2) \delta(y_2 - y_2) \langle \Delta n_p \Delta n_{\pi} \rangle \frac{\Delta p_T \Delta k_T}{\sigma_{inc}^2} \]

Then

\[ (F-1)_{yacc} = \frac{1}{N_{yacc}} \int dy_1 \int dy_2 C_{p_T}(y_1, y_2) \]

to simplify boost invariance: \( C_{p_T}(y_1, y_2) = C_{p_T}(y_1 - y_2) \)

and

\[ (F-1)_{yacc} = \int \frac{dy}{N_{yacc}} \left[ 2 \left( 1 - \frac{y}{y_{acc}} \right) C_{p_T}(y) \right] \]

Two limits of \( y_{acc} \):

\[ \frac{1}{dy_{acc}} \left\{ \begin{array}{ll} C_{p_T}(0) : y_{acc} \ll y_{corr} \\ \int dy C_{p_T}(y) : y_{acc} \gg y_{corr} \end{array} \right\} \]

Experiments A and B:

\[ \frac{(F-1)_A}{(F-1)_B} = \begin{cases} \frac{y_A}{y_B} : y_{A,B} \ll y_{corr} \\ 1 : y_{A,B} \gg y_{corr} \end{cases} \]

FRAMEWORK

ACCEPTANCE (rapidity)

\[ \frac{1}{N_{yacc}} \int dy \left[ \int dy_2 C_{p_T}(y_1, y_2) \right] \]

i.e. count particles with "weight" \( \frac{\Delta p_T \Delta k_T}{\sigma_{inc}^2} \)

"INTENSIVE"
(Frozen) Charge fluctuations

Asakawa, Heinz, Müller
Jeon, Koch

Fluctuations which do not have time to equilibrate in hadronic phase

Conserved quantity, such as \( q \).

\[ \langle \Delta q^2 \rangle_{\text{hadron}} \approx 2 - 3 \langle \Delta q^2 \rangle_{\text{qap}} \text{ (per same entropy)} \]

\( q(\text{quark}) < q(\text{pion}) \)

\( q \) conserved \( \Rightarrow \) can change only by diffusion

(Equation for harmonics of charge distribution in \( y \)

\[ f_k = \delta k^2 f_k + \text{noise} \quad k \approx \frac{1}{y} \]

\( \Rightarrow \) long wave harmonics of \( \Delta q \) relax slowly

\[ \Delta q^2 = \frac{\Sigma a_k^2 f_k^2}{k} \]

\( \Rightarrow \) for \( \Delta y \gg 1 \)

\[ \langle \Delta q^2 \rangle_{\text{present}} < \langle \Delta q^2 \rangle_{\text{hadron}} \]

Experiment (prelim): \( \langle \Delta q^2 \rangle_{\text{present}} = \langle q^2 \rangle_{\text{hadron}} \)

Conclusion: \( \Delta y \) not wide enough (diff. wins)?

Acceptance: analysis similar to \( p_T \) fluctuations: \( C_q(y) \)-balance function

What can we learn from E-b-e?

- By discovering critical point we map a distinct feature of the QCD phase diagram

- Using charge fluctuations we may be able to look back into the history of the collision, and see QGP.